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4D anomalous U(1)'s, their masses and their relation to 6D anomalies

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Abstract: In some four-dimensional orientifolds, U(1) gauge fields that are free of four-dimensional anomalies can still be massive. It is shown that this is due to mass-generating six-dimensional anomalies. Six-dimensional anomalies affect four-dimensional masses via decompactifications.

Keywords: Superstrings and Heterotic Strings, D-branes, Superstring Vacua, Anomalies in Field and String Theories.

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1. Introduction

Recently, many attempts have been made in order to embed the Standard Model (SM) in open string theory, with some success [1]–[7]. They consider the SM particles as open string states attached on different stacks of D-branes. N coincident D-branes typically generate a Unitary group U(N). Therefore, every stack of branes supplies the model with extra abelian gauge fields.

Such U(1) fields have generically four-dimensional anomalies. Such anomalies are cancelled via the Green-Schwarz mechanism [11, 12, 20] where a scalar axionic field (zero-form, or its dual two-form) is responsible for the anomaly cancellation. This mechanism gives a mass to the anomalous U(1) fields and breaks the associated gauge symmetry.

If the string scale is around a few TeV, observation of such anomalous U(1) gauge bosons becomes a realistic possibility [8, 9, 10].

As it has been shown in [16], we can compute the masses of the anomalous U(1)s by evaluating the ultraviolet tadpole of the one-loop open string diagram with the insertion of two gauge bosons on different boundaries. In this limit, the diagrams of the annulus with both gauge bosons in the same boundary and the Möbius strip do not contribute when vacua have cancelled tadpoles.

It turns out that U(1) gauge fields that are free of four-dimensional anomalies can still be massive [2, 15, 16]. We will show that this is due to the presence of mass-generating six-dimensional anomalies. Since there are decompactification limits in the theory, six-dimensional anomalies affect four-dimensional masses.

In six dimensions, two type of fields are necessary to cancel the anomalies, a scalar axion and a two-form. There is also a four-form field but it is dual to the scalar. Via the Green-Schwarz mechanism, the pseudoscalar axions give mass to the anomalous U(1) fields. However, the two-forms are not involved in mass generation.

In this paper, we show that four-dimensional non-anomalous U(1)s can have masses if their decompactification limits suffer by six-dimensional anomalies. We calculate the masses of the anomalous U(1)s of various six-dimensional orientifolds and we compare our results with decompactification limits of the four-dimensional orientifolds Z_6' and Z_6.

The paper is organized as follows. In section 2, we describe the structure of the six-dimensional mixed gauge anomalies. In section 3, we present the one-loop string computation for the mass formula of the anomalous U(1) in six dimensions. After, we give some examples of N=1 six-dimensional orientifolds where we provide the effective field theory predictions about the anomalous U(1)s and we evaluate the mass of these anomalous U(1)s using the formulas that we found before. In section 4, various decompactification limits of four-dimensional orientifold Z_6' and Z_6 are studied and compared with six-dimensional orientifolds.

2. The structure of six-dimensional mixed gauge anomalies

In six dimensions, the leading diagram that can give a contribution to anomalies is the square diagram (it has 1+D/2 external gauge bosons) [12]. In the presence of an anomalous

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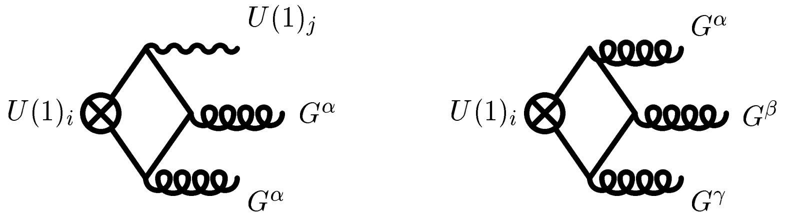

Figure 1: The anomalous diagrams are squares in six dimensions. The only mixed-gauge diagrams that are anomalous are $\mathrm{Tr}[Q_i Q_j T^\alpha T^\alpha]$ and $\mathrm{Tr}[Q_i T^\alpha T^\beta T^\gamma]$.

U(1) field, the effective action is not invariant under a transformation $\delta A^i = d\epsilon^i$. In six dimensions, the only possible non-zero mixed-anomaly diagrams are:

$$\delta S|_{gauge} = \int d^6x \left[\mathrm{Tr}[Q_i Q_j T^\alpha T^\alpha]\ \epsilon^i\ F^j \wedge \mathrm{Tr}[G^2] + \mathrm{Tr}[Q_i T^\alpha \{T^\beta T^\gamma\}]\ \epsilon^i\ \ \mathrm{Tr}[G^3] \right] \tag{2.1}$$

where powers of forms are understood as wedge products. We denote by G the field strength of a non-abelian gauge field W_μ. Gauge invariance is preserved by some other terms in the effective action that cancel the anomalous variations. The cancellation of the first anomalous term is arranged by a 2-form B^i which transform under the U(1) transformation like $\delta B^i = -\epsilon^i F^i$:

$$S_{QQTT} = \int d^6x \left[-\frac{1}{4g_i^2} F^{i2}_{\mu\nu} - \frac{1}{12} \left[dB^i + \Omega_{A^i} \right]^2 + C_1\ B^i \wedge \mathrm{Tr}[G^2] \right] \tag{2.2}$$

where $C_1 = \mathrm{Tr}[Q_i Q_j T^\alpha T^\alpha]$ is the anomaly of the first diagram and the 3-form $\Omega_A = AdA$ is the Chern-Simons term of the abelian gauge field A^i_μ. This part of the action does not generate a mass for the gauge boson.

By the (2.2), we can evaluate the action in terms of the dual 2-form λ of B [9]. Using $\mathrm{Tr}[G_i \tilde{G}_i] = d\Omega_{W_i}$, where $\Omega_W = \mathrm{Tr}[WdW + \frac{2}{3}W^3]$ is the Chern-Simons term for the non-abelian gauge field W^i, we finally find:

$$\tilde{S}_{QQTT} = \int d^6x \left[-\frac{1}{4g_i^2} F^{i2}_{\mu\nu} - \frac{1}{12} \left[d\lambda^i - 6C_1\ \Omega_{W^i} \right]^2 - \frac{1}{6} \Omega_{A^i} \wedge (d\lambda^i - 6C_1\ \Omega_{W^i}) \right]. \tag{2.3}$$

The λ^i are invariant under U(1) gauge transformations and transform like $\delta\lambda^i = 6C_1\,\mathrm{Tr}[G\epsilon^i]$ under a non-abelian gauge transformation $\delta W_\mu = D_\mu \epsilon$ so that the action is gauge invariant.

Thus, under a U(1) gauge transformation the variation of $\Omega_{A^i} \wedge d\lambda^i$ (since $\delta\Omega_{A^i} = d\epsilon F$) vanishes due to integration by parts and the term $C_1\ \Omega_{A^i} \wedge \Omega_{W^i}$ cancels the first anomaly in (2.1).

The second anomaly is cancelled by a pseudoscalar axion that transforms under the U(1) transformation as $\delta\alpha^i = -\epsilon^i$:

$$S_{QTTT} = \int d^6x \left[-\frac{1}{4g_i^2} F^{i2}_{\mu\nu} + \frac{M^2}{2} (A^i + d\alpha^i)^2 + C_2\ \alpha^i\ \ \mathrm{Tr}[G^3] \right] \tag{2.4}$$

where $C_2 = \mathrm{Tr}[Q_i T^\alpha \{T^\beta T^\gamma\}]$ is the anomaly of the second diagram. This action supplies a mass term for the U(1) gauge field and breaks the gauge symmetry in six dimensions.

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3. Calculation of the mass of the anomalous U(1)s for six-dimensional orientifolds

In this section we will evaluate the contribution to the anomalous U(1) mass for six-dimensional supersymmetric orientifolds. These models appear as decompactification limits of four-dimensional orientifolds.

In type-I string theory, the axions that are relevant for anomaly cancellation come from the RR sectors. The mass-term in (2.4) is coming from different orders of string perturbation theory. The $(\partial\alpha^i)^2$ is a tree-level (sphere) term, the $A^i\partial\alpha^i$ comes in the disk and the quadratic term in the gauge fields is a one-loop contribution. To clarify this, we mention that g_i^2 is proportional to $g_s = e^\phi$ and every power of the axion absorbs a dilaton factor $e^{-\phi}$ because it is a RR filed. The string perturbation series are weighted by $g_s^{-\chi}$ where $\chi = 2 - 2h - c - b$ is the Euler character and h, c and b denote the handle, the cross-cups and the boundaries of a closed orientable Riemann surface respectively.

The diagrams at one-loop that contribute to terms quadratic in the gauge bosons (anomalous U(1)s) are the genus-one surfaces with boundaries: the annulus and the Möbius strip. In the infrared (IR) region they diverge logarithmically and give the logarithmic running of the couplings. In the ultraviolet (UV) region the tadpoles of the annulus with both gauge bosons inserted in the same boundary and the Möbious strip vanish due to the tadpole cancellation. However, in this UV limit the annulus amplitude with the gauge bosons inserted in opposite boundaries provides the mass-term of the anomalous U(1) [16]. Since we are interested in the anomalous gauge boson mass, we concentrate on the latter diagram. The gauge boson vertex operator is

$$\tilde{V}^a = \lambda^a \epsilon_\mu (\partial X^\mu + i(p\cdot\psi)\psi^\mu) e^{ip\cdot X} \tag{3.1}$$

where λ is the Chan-Paton matrix and ϵ^μ is the polarization vector. The 2-point annulus amplitude is given by

$$\mathcal{A}^{ab} = -\frac{1}{4G}\int [d\tau][dz] \int \frac{d^6p}{(2\pi)^6} \sum_k \left\langle \tilde{V}^a(\epsilon_1, p_1, z)\tilde{V}^b(\epsilon_2, p_2, z_0)\right\rangle_k \tag{3.2}$$

where G denotes the order of the orientifold group. The fundamental polygon of the annulus is $[0, t/2]\otimes[0, 1/2]$. The index k denotes the various orbifold sectors that we may have. Using the translation symmetry of the annulus, we fix the position of one VO to $z_0 = 1/2$. The other VO is placed on the imaginary axis with $z \in [0, t/2]$.

The leading term of (3.2) is

$$\mathcal{A}^{ab} = \int \frac{d^6p}{(2\pi)^6}[(\epsilon_1\cdot\epsilon_2)(p_1\cdot p_2) - (\epsilon_1\cdot p_2)(p_1\cdot\epsilon_2)] \sum_k \mathrm{Tr}[\gamma_k\lambda^\alpha]\,\mathrm{Tr}[\gamma_k\lambda^\beta]\mathcal{A}_k^{ab}\,. \tag{3.3}$$

where

$$\mathcal{A}_k^{ab} = -\frac{1}{2G}\int [d\tau][dz] e^{-p_1\cdot p_2\langle X(z)X(1/2)\rangle}\left[\left\langle \psi(z)\psi\left(\frac{1}{2}\right)\right\rangle^2 - \left\langle X(z)\partial X\left(\frac{1}{2}\right)\right\rangle^2\right] Z_k^{ab}\,. \tag{3.4}$$

since the p-independent terms vanish due to supersymmetry. The bosonic and fermionic correlation functions are given in the appendix B.4, B.5.

It appears that the amplitude (3.2) has a kinimatical multiplicative factor that is $\mathcal{O}(p^2)$, thus would seem to provide a leading correction only to the anomalous gauge boson coupling. We will see however, that after integration over the position z and the annulus modulus τ_2, a term proportional to $1/p_1 \cdot p_2$ appears from the ultraviolet (UV) region (as a result of the quadratic UV divergence in the presence of anomalous U(1)s) that will provide the mass-term.

Strictly speaking, the amplitude above is zero on-shell if we enforce the physical state conditions $\epsilon \cdot p = p^2 = 0$ and momentum conservation $p_1 + p_2 = 0$. There is however a consistent off-shell extension, without imposing momentum conservation, that has given consistent results in other cases (see [22] for a discussion) and we adopt it here. We will thus impose momentum conservation only at the end of the calculation.

Spin structure summation of the partition function Z_k^{ab}, gives zero due to space-time supersymmetry. Therefore, terms in the correlation functions which are spin-structure independent vanish. The only spin-dependant term lies in the fermionic correlation function:

$$\left\langle \psi\left(z-\frac{1}{2}\right)\psi(0)\right\rangle^2 [^{\alpha}_{\beta}]_{\rm annulus} = -2\pi i\partial_\tau \log\vartheta[^{\alpha}_{\beta}](0|\tau)\,. \tag{3.5}$$

Eq. (3.5) is independent of z, the position of the second VO. Thus, we can easily integrate on dz. Using the modular transformations of the theta functions and keeping the leading order of δ, we have:

$$\int_0^{\tau_2} dz\; e^{-\delta\langle X(z)X(0)\rangle} = \int_0^{\tau_2} dz\; \tau_2^{\delta/2}\frac{(2\pi\eta^3(\tau))^\delta}{\vartheta[^0_1](z/\tau|-1/\tau)} = \tau_2^{1+\delta/2}[2\pi\eta^3(\tau)]^\delta + \cdots\,. \tag{3.6}$$

Following the procedure of [16] we rewrite (3.4) as:

$$\mathcal{A}_k^{ab} = -\frac{1}{2G}\int [d\tau]\tau_2^{1+\delta/2}[2\pi\eta^3(\tau)]^\delta F_k^{ab}\,. \tag{3.7}$$

defining F_k^{ab} as a term which contains all the spin-structure and the orbifold information:

$$F_k^{ab} = \sum_{\alpha\beta}\eta^{\alpha\beta}\left[-2\pi i\partial\log\vartheta[^{\alpha}_{\beta}]\right]\left[\frac{1}{(2\pi\tau)^3}\frac{\vartheta[^{\alpha}_{\beta}]^2}{\eta^6}\right] Z_{int,k}^{ab}[^{\alpha}_{\beta}] \tag{3.8}$$

where $\eta^{\alpha\beta} = \frac{1}{2}(-1)^{\alpha+\beta+\alpha\beta}$. The first bracket is denoting the VO insertion in the annulus diagram. The second is the six-dimensional partition function.

3.1 Six-dimensional N=1 orientifolds examples

Usual six-dimensional decompactification limits of four-dimensional supersymmetric orientifolds are the N=1 orientifolds of type-IIB string theory, $\mathbb{R}^6 \times K3/Z_N$ where the only possible choices are $N = 2, 3, 4, 6$. Thus, we will apply the above general formulae on these orientifolds.

We re-evaluate the massless spectrum of these models using the 'shift' vectors that are given for each model. The result is provided in table 1. We were especially careful in distinguishing the representations from the conjugate representations since this was not

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Twist Group / Gauge Group	(99)/(55) matter	(95) matter
Z_2 / $\mathrm{U}(16)_9 \times \mathrm{U}(16)_5$	$2 \times 120 + 2 \times \overline{120}$	$(16; \overline{16}) + (\overline{16}; 16)$
Z_3 / $\mathrm{U}(8) \times \mathrm{SO}(16)$	$(8, 16_v) + (\overline{8}, 16_v)$ $+(28, 1) + (\overline{28}, 1)$	-
Z_4 / $\mathrm{U}(8)_9 \times \mathrm{U}(8)_9 \times$ $\mathrm{U}(8)_5 \times \mathrm{U}(8)_5$	$(28, 1) + (\overline{28}, 1)$ $+(1, 28) + (1, \overline{28})$ $+(8, \overline{8}) + (\overline{8}, 8)$	$(8, 1; \overline{8}, 1) + (\overline{8}, 1; 8, 1)$ $+(1, 8; 1, \overline{8}) + (1, \overline{8}; 1, 8)$
Z_6 / $(\mathrm{U}(4)^2 \times \mathrm{U}(8))_9 \times$ $+(\mathrm{U}(4)^2 \times \mathrm{U}(8))_5$	$(\underline{6, 1}, 1) + (\underline{\overline{6}, 1}, 1)$ $+(\underline{4, 1}, \overline{8}) + (\underline{\overline{4}, 1}, 8)$	$(4, 1, 1; \overline{4}, 1, 1) + (\overline{4}, 1, 1; 4, 1, 1)$ $+(1, 4, 1; 1, \overline{4}, 1) + (1, \overline{4}, 1; 1, 4, 1)$ $+(1, 1, 8; 1, 1, \overline{8}) + (1, 1, \overline{8}; 1, 1, 8)$

Table 1: The transformations of the massless fermionic states in all the D=6 orientifolds. The underlined numbers denote all the possible permutations.

SU(N) Representation	Cubic Casimir	U(1) Charge
$\square$	$A(\square) = 1$	$Q(\square) = 1$
$\overline{\square}$	$A(\overline{\square}) = -1$	$Q(\overline{\square}) = -1$
$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	$A(\begin{smallmatrix}\square\\\square\end{smallmatrix}) = N - 4$	$Q(\begin{smallmatrix}\square\\\square\end{smallmatrix}) = 2$
$\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}$	$A(\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}) = -N + 4$	$Q(\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}) = -2$

Table 2: Our normalization of the cubic casimir and the U(1) charge of the SU(N) representations.

transparent in the previous literature [17, 18, 19]. Tadpole cancellation guarantees that the models are free of irreducible non-abelian anomalies [21, 12]. This is also shown in appendix D.

The mixed-anomaly traces can be easily evaluated for each orientifold. Our normalization of the cubic casimir and the U(1) charge of the SU(N) representations are given in table 2.

The general mass formulae for the anomalous U(1) gauge fields in the orientifolds above can be easily evaluated. More details for the explicit computations of the UV tadpole in the annulus diagram (3.2) are provided in the appendix C. The results for strings attached on the same kind of branes (untwisted states) are (C.3)

$$\frac{1}{2} M_{aa}^2 = -\frac{4}{\pi^2 N} \sum_k \sin^2 \frac{\pi k}{N} \; \mathrm{Tr}[\gamma_k \lambda^a] \, \mathrm{Tr}[\gamma_k \lambda^a] \tag{3.9}$$

where $a = 5, 9$ denotes the kind of D-branes on which the open string is attached. In the case where strings have one end on a $D5$ and the other on a $D9$-brane (twisted states) we have:[1]

$$\frac{1}{2} M_{59}^2 = -\frac{1}{\pi^2 N} \sum_k \mathrm{Tr}[\gamma_k \lambda^5] \, \mathrm{Tr}[\gamma_k \lambda^9] \,. \tag{3.10}$$

[1]We could end up in the same results (3.9), (3.10) if we made use of the axionic couplings for the six-dimensional orientifolds, evaluated first for the Z_2 case in [23] and after for all Z_N in [24].

We should mention, that the above masses are unormalized. To obtain the normalized mass matrix, we must also take into account the kinetic terms of the U(1) gauge bosons which are

$$S_{\text{kinetic}} = -\frac{1}{4g_s}\left[\mathcal{V}_1\mathcal{V}_2\sum_i F_i^2 + \sum_j \tilde{F}_j^2\right]. \tag{3.11}$$

where i and j denote the gauge groups that are coming from different stacks of D9 and D5-branes. This implies $M_{99}^2 \to M_{99}^2/(\mathcal{V}_1\mathcal{V}_2)$, $M_{55}^2 \to M_{55}^2$ and $M_{95}^2 \to M_{95}^2/(\sqrt{\mathcal{V}_1\mathcal{V}_2})$.

A convenient formula of the action of the orbifold elements on the Chan-Paton factors is defined by the matrices:

$$\gamma_k = e^{-2\pi i k V\cdot H} \tag{3.12}$$

where H_I the Cartan generators of SO(32), represented by diagonal matrices having the σ^3 Pauli matrix in the diagonal and everywhere else zero. Notice that the normalization of the Cartans is $\text{Tr}[H_I^2] = 2$. V^I is a 16-dimensional "shift" vector, defined by (3.12) and satisfies the tadpole conditions for each orientifold.

The normalized generators of the anomalous $\text{U}(1)_i$ are defined as:

$$\lambda_i^\alpha = \frac{1}{2\sqrt{n_i}}\sum Q_i^\alpha \cdot H \tag{3.13}$$

where α denotes the type of brane. The $Q_i^\alpha = (0,\ldots,0,1,\ldots,1,0,\ldots,0)$ is a 16-dimensional vector with n_i entries of 1s where the $\text{SU}(n_i-1)$ lives. We normalize the λ matrices with $\text{Tr}[\lambda^2] = 1/2$. Thus, the relevant trace is:

$$\text{Tr}[\gamma_k^\alpha \lambda_i^\alpha] = \text{Tr}[e^{-2\pi i k V^\alpha\cdot H} Q_i^\alpha \cdot H] = -\frac{i}{\sqrt{n_i}}\sin[2\pi k V_i^\alpha] \tag{3.14}$$

where V_i^α are the overlapping components of V^α and Q^α [20].

3.1.1 Z_2 orientifold

For the Z_2, the tadpole condition gives 32 $D9$ and 32 $D5$-branes [17, 18, 19]. The characteristic vectors are:

$$V_{5,9} = \frac{1}{4}(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1)\,. \tag{3.15}$$

The gauge group is $\text{U}(16)_9 \times \text{U}(16)_5$. The massless states are given in table 1 and we use them to evaluate the mixed anomalous diagrams. We are interested in anomalous diagrams with one abelian and three non-abelian gauge bosons $\text{U}(1)\times\text{SU}(N)^3$ since their cancellation provides the six-dimensional mass-term. We find:

$$A_{QTTT} = 32\cdot\begin{pmatrix} 4 & -1 \\ -1 & 4\end{pmatrix}. \tag{3.16}$$

where the columns label the U(1)s, while the rows label the non-abelian factors. The matrix has two non-zero eigenvalues and both anomalous $U(1)$s are expected to become massive [24]. The unormalized mass matrix for the anomalous $U(1)$s is calculated by the use of (3.9), (3.10) and (3.14):

$$\frac{1}{2}M^2 = -\frac{1}{2\pi^2}\begin{pmatrix} 4\,\mathrm{Tr}[\gamma_1\lambda^9]\,\mathrm{Tr}[\gamma_1\lambda^9] & \mathrm{Tr}[\gamma_1\lambda^9]\,\mathrm{Tr}[\gamma_1\lambda^5] \\ \mathrm{Tr}[\gamma_1\lambda^5]\,\mathrm{Tr}[\gamma_1\lambda^9] & 4\;\mathrm{Tr}[\gamma_1\lambda^5]\,\mathrm{Tr}[\gamma_1\lambda^5] \end{pmatrix} = \frac{8}{\pi^2}\begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}. \tag{3.17}$$

As it was expected from the effective field theory computation of the anomalies, there are two massive eigenstates: $\pm A + \tilde{A}$ with masses $24/\pi^2$, $40/\pi^2$ (we denote with A the gauge boson that is coming from the $D9$-branes and with $\tilde{A}$ the one that is coming from the $D5$).

3.1.2 Z_3 orientifold

The Z_3 orientifold does not contain a Z_2 reflection element. Thus, there are no $D5$-branes. The characteristic vector is:

$$V_9 = \frac{1}{3}(1,1,1,1,1,1,1,1,0,0,0,0,0,0,0,0) \tag{3.18}$$

and the gauge group $\mathrm{U}(8)\times\mathrm{SO}(16)$. From the massless spectrum which is provided in table 1 we find that the single gauge boson suffers from mixed non-abelian anomalies [24].

$$A_{QTTT} = 48\,. \tag{3.19}$$

Using (3.14) we find the mass of this gauge boson:

$$\frac{1}{2}M^2 = \frac{32}{3\pi^2}\sum_{k=1}^{2}\sin^2\frac{\pi k}{3}\sin^2\frac{2\pi k}{3} = \frac{12}{\pi^2}\,. \tag{3.20}$$

3.1.3 Z_4 orientifold

The Z_4 orientifold contains 32 $D9$-branes and 32 $D5$-branes. The characteristic vectors are:

$$V_{5,9} = \frac{1}{8}(1,1,1,1,1,1,1,1,3,3,3,3,3,3,3,3) \tag{3.21}$$

and the gauge group is $\mathrm{U}(8)_9\times\mathrm{U}(8)_9\times\mathrm{U}(8)_5\times\mathrm{U}(8)_5$. The $\mathrm{U}(1)\times\mathrm{SU}(N)^3$ anomalies are:

$$A_{QTTT} = 16\cdot\begin{pmatrix} 3 & -1 & -1 & 0 \\ -1 & 3 & 0 & -1 \\ -1 & 0 & 3 & -1 \\ 0 & -1 & -1 & 3 \end{pmatrix}. \tag{3.22}$$

where again the columns label the U(1)s and the rows the non-abelian factors $\mathrm{SU}(8)_9^2\times\mathrm{SU}(8)_5^2$. Notice that we have two equal matrices in the diagonal blocks and two other ones equal in the off-diagonal blocks. This is a consequence of the fact that the $D9$ and $D5$ branes are related by T-duality and split in isomorphic groups. All those models are T-selfdual.[2] The anomaly matrix has four non-zero eigenvalues [24].

[2]Except from the Z_3 orientifold which is T-dual to a Z_6' model that does not contain the pure Ω element [18].

The mass matrix of the anomalous U(1) masses is

$$\frac{1}{2}M^2 = \frac{4}{\pi^2}\begin{pmatrix} 3 & -1 & 1 & 0 \\ -1 & 3 & 0 & 1 \\ 1 & 0 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{pmatrix}. \tag{3.23}$$

Diagonalizing this matrix, we find four massive U(1) fields that are in accordance with the anomalies. The massive U(1) fields are $-A_1 - A_2 + \tilde{A}_1 + \tilde{A}_2$, $A_1 + \tilde{A}_2$, $A_2 + \tilde{A}_1$, $-A_1 + A_2 - \tilde{A}_1 + \tilde{A}_2$ with masses $4/\pi^2$, $12/\pi^2$, $12/\pi^2$, $20/\pi^2$ respectively.

3.1.4 Z_6 orientifold

The Z_6 orientifold contains 32 $D9$-branes and 32 $D5$-branes. The characteristic vectors are:

$$V_{5,9} = \frac{1}{12}(1,1,1,1,5,5,5,5,3,3,3,3,3,3,3,3) \tag{3.24}$$

and the gauge group $\mathrm{U}(4)_9 \times \mathrm{U}(4)_9 \times \mathrm{U}(8)_9 \times \mathrm{U}(4)_5 \times \mathrm{U}(4)_5 \times \mathrm{U}(8)_5$.

The $\mathrm{U}(1) \times \mathrm{SU}(N)^3$ anomalies are:

$$A_{QTTT} = 8 \cdot \begin{pmatrix} 3 & 0 & -2 & -1 & 0 & 0 \\ 0 & 3 & -2 & 0 & -1 & 0 \\ -1 & -1 & 4 & 0 & 0 & -2 \\ -1 & 0 & 0 & 3 & 0 & -2 \\ 0 & -1 & 0 & 0 & 3 & -2 \\ 0 & 0 & -2 & -1 & -1 & 4 \end{pmatrix}. \tag{3.25}$$

The columns are the U(1)s and the rows the non-abelian factors, always in the ordered form of table 1. The (3.25) has five non-zero and one zero eigenvalue which corresponds to $A_1 + A_2 + A_3 + \tilde{A}_1 + \tilde{A}_2 + \tilde{A}_3$. Our result is in accordance with [24] where it had been shown that one of the six U(1) factor remains unbroken. The independent axions that participate in the cancellation of the anomaly and the mass generation are only five.

The mass matrix for the anomalous U(1)s is

$$\frac{1}{2}M^2 = \frac{2}{\pi^2}\begin{pmatrix} 3 & 0 & -\sqrt{2} & 1 & 0 & 0 \\ 0 & 3 & -\sqrt{2} & 0 & 1 & 0 \\ -\sqrt{2} & -\sqrt{2} & 4 & 0 & 0 & 2 \\ 1 & 0 & 0 & 3 & 0 & -\sqrt{2} \\ 0 & 1 & 0 & 0 & 3 & -\sqrt{2} \\ 0 & 0 & 2 & -\sqrt{2} & -\sqrt{2} & 4 \end{pmatrix}. \tag{3.26}$$

Diagonalizing the mass matrix, we find that five U(1) fields become massive and one remains massless. The effective field theory computation agrees with the result above.

4. The decompactification limits of four-dimensional $N = 1$ orientifolds

We are interested in studying the decompactification limits of four-dimensional orientifolds, in order to investigate potential six-dimensional anomalies. In this section, we focus on the Z_6' and Z_6 four-dimensional orientifold since they have enought of structure needed. The four-dimensional spectra of these models are provided in table 3.

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Twist Group / Gauge Group	(99)/(55) matter	(95) matter
Z'_6	$(\overline{4},1,8)+(1,4,\overline{8})+(6,1,1)$	$(\overline{4},1,1;\overline{4},1,1)+(1,4,1;1,4,1)$
$\mathrm{U}(4)_9^2\times\mathrm{U}(8)_9\times$	$+(1,\overline{6},1)+(4,1,8)+(1,\overline{4},\overline{8})$	$+(1,\overline{4},1;1,1,8)+(1,1,8;1,\overline{4},1)$
$\mathrm{U}(4)_5^2\times\mathrm{U}(8)_5$	$+(\overline{4},4,1)+(4,4,1)+(\overline{4},\overline{4},1)$	$+(4,1,1;1,1,\overline{8})+(1,1,\overline{8};4,1,1)$
	$+(1,1,28)+(1,1,\overline{28})$	
Z_6	$2(15,1,1)+2(1,\overline{15},1)$	$(6,1,1;6,1,1)+(1,\overline{6},1;1,\overline{6},1)$
$\mathrm{U}(6)_9^2\times\mathrm{U}(4)_9\times$	$+2(\overline{6},1,4)+2(1,6,\overline{4})$	$+(1,6,1;1,1,\overline{4})+(1,1,\overline{4};1,6,1)$
$\mathrm{U}(6)_5^2\times\mathrm{U}(4)_5$	$+(\overline{6},1,\overline{4})+(1,6,4)+(6,\overline{6},1)$	$+(\overline{6},1,1;1,1,4)+(1,1,4;\overline{6},1,1)$

Table 3: The transformations of the massless fermionic states in two D=4 orientifolds.

4.1 The four-dimensional Z'_6 orientifold

The orbifold rotation vector is $(v_1, v_2, v_3) = (1, -3, 2)/6$. Since there is an order two twist ($k = 3$), we have one set of $D5$-branes. Tadpole cancellation implies the existence of 32 $D9$-branes and 32 $D5$-branes that we put together at one of the fixed points of the Z_2 action (namely the origin). The Chan-Paton 'shift' vectors are

$$V_{5,9} = \frac{1}{12}(1,1,1,1,5,5,5,5,3,3,3,3,3,3,3,3)\,. \tag{4.1}$$

The gauge group has a factor of $\mathrm{U}(4)\times\mathrm{U}(4)\times\mathrm{U}(8)$ coming from the $D9$-branes and an isomorphic factor coming from the $D5$-branes. Different sectors preserve different supersymmetries. The $N = 1$ sectors correspond to $k = 1, 5$, while for $k = 2, 3, 4$ we have $N = 2$ sectors.

The four-dimensional anomalies of the U(1)s have been computed in [20] and the anomaly matrix is

$$A_{QTT} \sim \begin{pmatrix} 2 & 2 & 4\sqrt{2} & -2 & 0 & -2\sqrt{2} \\ -2 & -2 & -4\sqrt{2} & 0 & 2 & 2\sqrt{2} \\ 0 & 0 & 0 & 2 & -2 & 0 \\ -2 & 0 & -2\sqrt{2} & 2 & 2 & 4\sqrt{2} \\ 0 & 2 & 2\sqrt{2} & -2 & -2 & -4\sqrt{2} \\ 2 & -2 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{4.2}$$

there are two linear combinations that are free of four-dimensional anomalies: $\sqrt{2}(A_1 + A_2) - A_3$ and $\sqrt{2}(\tilde{A}_1 + \tilde{A}_2) - \tilde{A}_3$.

The contribution to the mass matrix [16] is

$$\begin{aligned} \frac{1}{2}M^2_{aa,ij} = &-\frac{\sqrt{3}}{24\pi^3}\left(\mathrm{Tr}[\gamma_1\lambda_i^a]\,\mathrm{Tr}[\gamma_1\lambda_j^a] + \mathrm{Tr}[\gamma_5\lambda_i^a]\,\mathrm{Tr}[\gamma_5\lambda_j^a]\right) - \\ &-\frac{1}{4\pi^3}\left(\mathcal{V}_2\delta_{a,9} + \frac{1}{4\mathcal{V}_2}\delta_{a,5}\right)\left(\mathrm{Tr}[\gamma_2\lambda_i^a]\,\mathrm{Tr}[\gamma_2\lambda_j^a] + \mathrm{Tr}[\gamma_4\lambda_i^a]\,\mathrm{Tr}[\gamma_4\lambda_j^a]\right) - \\ &-\frac{\mathcal{V}_3}{3\pi^3}\,\mathrm{Tr}[\gamma_3\lambda_i^a]\,\mathrm{Tr}[\gamma_3\lambda_j^a] \end{aligned} \tag{4.3}$$

for $a = 5, 9$ where $\delta_{a,b}$ is the Kronecker delta. The mixed 59 annulus diagrams give a contribution to the mass

$$\begin{aligned}\frac{1}{2}M^2_{95,ij} &= -\frac{\sqrt{3}}{48\pi^3}\Big(\,\mathrm{Tr}[\gamma_1\lambda^9_i]\,\mathrm{Tr}[\gamma_1\lambda^5_j] + \mathrm{Tr}[\gamma_5\lambda^9_i]\,\mathrm{Tr}[\gamma_5\lambda^5_j] + \\ &\qquad + \mathrm{Tr}[\gamma_2\lambda^9_i]\,\mathrm{Tr}[\gamma_2\lambda^5_j] - \mathrm{Tr}[\gamma_4\lambda^9_i]\,\mathrm{Tr}[\gamma_4\lambda^5_j]\Big) - \\ &\quad - \frac{\mathcal{V}_3}{12\pi^3}\,\mathrm{Tr}[\gamma_3\lambda^9_i]\,\mathrm{Tr}[\gamma_3\lambda^5_j]\,. \end{aligned} \tag{4.4}$$

The unormalized mass matrix [16] has eigenvalues and eigenvectors:

$$m_1^2 = 6\mathcal{V}_2\,, \qquad -A_1 + A_2 \tag{4.5}$$

$$m_2^2 = \frac{3}{2\mathcal{V}_2}\,, \qquad -\tilde{A}_1 + \tilde{A}_2 \tag{4.6}$$

$$m_{3,4}^2 = \frac{5\sqrt{3} + 48\mathcal{V}_3 \pm \sqrt{3}\alpha}{12}\,, \qquad \frac{-3 \pm \alpha}{4\sqrt{2}(4\sqrt{3}\mathcal{V}_3 - 1)}(A_1 + A_2 - \tilde{A}_1 - \tilde{A}_2) - A_3 + \tilde{A}_3\,; \tag{4.7}$$

$$m_{5,6}^2 = \frac{15\sqrt{3} + 80\mathcal{V}_3 \pm \beta}{12}\,, \qquad \frac{9\sqrt{3} \mp \beta}{4\sqrt{2}(20\mathcal{V}_3 - 3\sqrt{3})}(A_1 + A_2 + \tilde{A}_1 + \tilde{A}_2) + A_3 + \tilde{A}_3\,; \tag{4.8}$$

with $\alpha = \sqrt{25 - 128\sqrt{3}\mathcal{V}_3 + 768\mathcal{V}_3^2}$ and $\beta = \sqrt{5(135 - 384\sqrt{3}\mathcal{V}_3 + 1280\mathcal{V}_3^2)}$. Note that the eigenvalues are invariant under the T-duality symmetry of the theory $\mathcal{V}_2 \to 1/4\mathcal{V}_2$. Thus, all U(1)s become massive, including the two anomaly free combinations.

4.2 Decompactification of the Z_6' orientifold

The axions that cancel the anomalies, being twisted RR fields, are localized on the fixed points of the internal dimensions. Since there are various orbifold sectors k, there are also various axions α_k^i localized on the fixed points of the internal tori where the k-th orbifold element acts [14]. Thus, in the Z_6' orientifold, the α_1^i, α_5^i axions are living in the 4D Minkowski space, the α_2^i, α_4^i in 4D Minkowski space plus the second torus T_2^2 and the α_3^i in 4D Minkowski space plus the third torus T_3^2.

The decompactification limit of the first torus ($\mathcal{V}_1 \to \infty$) does not have any special interest since none of the fields become six-dimensional.

4.2.1 Decompactification of the second torus ($\mathcal{V}_2 \to \infty$)

If we decompactify the second torus ($\mathcal{V}_2 \to \infty$) the 99 states that are coming from the $k = 2, 4$ sectors and the α_2^i, α_4^i axions become 6 dimensional fields. The gauge group is enhanced and can be found by the action of γ_2, γ_4 on the Chan-Paton factors. The fields of the other sectors remain four-dimensional and do not contribute to six-dimensional anomalies. The 'shift' vector will be $2V_9$, where V_9 is given in (4.1). Following the known procedure we find that the four-dimensional U(4) $\times$ U(4) $\times$ U(8) gauge group is enhanced in U(8) $\times$ SO(16). The generators of the U(4)$_1$ $\times$ U(4)$_2$ are enhanced in the generators of the U(8) as $T_{\mathrm{U}(8)} \sim T_{\mathrm{U}(4)_1} \oplus \overline{T}_{\mathrm{U}(4)_2}$ and the generators of the U(8) in the generators of the SO(16).

The rest of the matter fields are combined with some Kaluza-Klein states, that now become massless, to give the representations of the greater gauge group. The (4, 4, 1),

$(\bar{4},\bar{4},1)$ are now contained in the adjoint of the U(8) as the $(1,1,28)$, $(1,1,\overline{28})$ are contained in the adjoint of the SO(16). The $(6,1,1)$, $(1,\bar{6},1)$ form the antisymmetric $(28,1)$. The $(\bar{4},4,1)$ form the $(\overline{28},1)$. Finally, the $(4,1,8)$, $(1,\bar{4},\bar{8})$, $(4,1,\bar{8})$ and $(1,\bar{4},8)$ form the bi-fundamental $(8,16)$. Thus, the effective gauge group is the one that it was taken from the Z_3 six-dimensional orientifold (table 1).

The spectrum of the Z_3 six-dimensional orientifold contains an anomalous gauge boson (chapter 3.1.2). By the way that the $\mathrm{U}(4)\times\mathrm{U}(4)\times\mathrm{U}(8)$ gauge group is enhanced in $\mathrm{U}(8)\times\mathrm{SO}(16)$, we find that the anomalous gauge boson is A_1-A_2 and becomes massive due to the six-dimensional Green-Schwarz mechanism. This mass can be evaluated by the six dimensional formulae and it is given in (3.20). The A_1+A_2 and A_3 are enhanced in the non-abelian factors and they have no anomalies.

The contribution of the six-dimensional masses to the four-dimensional ones can be found by taking the $\mathcal{V}_2\to\infty$ limit of (4.3):

$$\frac{1}{2}M^2_{99,ij}=-\frac{1}{4\pi^3}\left(\mathrm{Tr}[\gamma_2\lambda^9_i]\,\mathrm{Tr}[\gamma_2\lambda^9_j]+\mathrm{Tr}[\gamma_4\lambda^9_i]\,\mathrm{Tr}[\gamma_4\lambda^9_j]\right). \tag{4.9}$$

which is the same as the formula of the masses in the six-dimensional Z_3 orientifold (3.20) upon normalization. The sectors $k=2,4$ of the four-dimensional Z_6' orientifold in this limit are the $k=1,2$ sectors of the six-dimensional Z_3 orientifold. Using (3.13) and (4.1), we evaluate the mass-matrix of the anomalous U(1)s. The mass-matrix has two zero eigenvalues, with eigenvectors: A_3, A_1+A_2 and a massive state with eigenvalue:

$$-A_1+A_2\,,\qquad m^2=\frac{3}{\pi^3} \tag{4.10}$$

as it was expected by the way that the initial $\mathrm{U}(4)\times\mathrm{U}(4)\times\mathrm{U}(8)$ gauge group is enhanced in $\mathrm{U}(8)\times\mathrm{SO}(16)$. This six-dimensional contribution affects the four-dimensional mass (4.5).

The results confirm that anomalous gauge bosons in six-dimensions that become massive through the six-dimensional Green-Schwarz mechanism, contribute to the four-dimensional mass generation by a normalized term.

4.2.2 Decompactification of the third torus ($\mathcal{V}_3\to\infty$)

If we decompactify the third torus ($\mathcal{V}_3\to\infty$), all the string states from the $k=3$ sector and the a^i_3 axions become six-dimensional. The new gauge group can be found by the action of the γ_3 on the Chan-Paton. The orbifold rotation $3(v_1,v_2)=(1,-1)/2$ shows that D5-branes survive in this limit. The 'shift' vector is now $3V_a$ where V_a is given in(4.1). The four-dimensional $\mathrm{U}(4)_\alpha\times\mathrm{U}(4)_\alpha\times\mathrm{U}(8)_\alpha$ gauge group (where $\alpha=5,9$) is enhanced to $\mathrm{U}(16)_\alpha$ that is the gauge group of the Z_2 six-dimensional orientifold. The generators are $T_{\mathrm{U}(16)}\sim T_{\mathrm{U}(4)_1}\oplus T_{\mathrm{U}(4)_2}\oplus\overline{T}_{\mathrm{U}(8)}$. Therefore, $(1,\bar{4},\bar{8})_a$, $(4,1,8)_a$, $(\bar{4},4,1)_a$ are enhanced in the adjoint of the $\mathrm{U}(16)_a$. The $(6,1,1)_a$, $(1,4,\bar{8})_a$, $(1,1,\overline{28})_a$, $(4,4,1)_a$ form the antisymmetric 120_a. The $(\bar{4},1,8)_a$, $(\bar{4},\bar{4},1)_a$, $(1,1,28)_a$, $(1,\bar{6},1)_a$ are enhanced in the $\overline{120}$.

From the way that the generators are formed we can expect that the abelian factor of $\mathrm{U}(16)_9$, $A\sim A_1+A_2-\sqrt{2}A_3$ where the coefficients are coming from the normalization of the generators of different rank. Similarly for the abelian factor of $\mathrm{U}(16)_5$, $\tilde{A}\sim\tilde{A}_1+\tilde{A}_2-\sqrt{2}\tilde{A}_3$.

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As we have seen in section 3.1.1, the new gauge group contains two anomalous bosons in six dimensions which are linear combinations of the A and $\tilde{A}$. The other mass eigenstates are embedded in the non-abelian factors. The masses of the six-dimensional gauge bosons have been found in (3.17). The contribution of the six-dimensional mass-terms to the four-dimensional mass generation can be found by taking the $\mathcal{V}_3 \to \infty$ limit in (4.3), (4.4) and these are ($a = 5, 9$):

$$\frac{1}{2} M^2_{aa,ij} = -\frac{1}{3\pi^3} \operatorname{Tr}[\gamma_3 \lambda_i^a] \operatorname{Tr}[\gamma_3 \lambda_j^a] \,. \tag{4.11}$$

and for 59 states:

$$\frac{1}{2} M^2_{59,ij} = -\frac{1}{12\pi^3} \operatorname{Tr}[\gamma_3 \lambda_i^5] \operatorname{Tr}[\gamma_3 \lambda_j^9]. \tag{4.12}$$

which are the same (upon normalization) with the contributions of the six-dimensional generation of the Z_2 orientifold (section 3.1.1). In this limit, the $k = 3$ sector of the six-dimensional Z_6' orientifold is the $k = 1$ sector of the six-dimensional Z_2 one. The mass-matrix has four zero eigenvalues, with eigenvectors: $\sqrt{2}\tilde{A}_1 + \tilde{A}_3$, $-\tilde{A}_1 + \tilde{A}_2$, $\sqrt{2}A_1 + A_3$, $-A_1 + A_2$ and two massive states with eigenvalues:

$$\begin{aligned} A_1 + A_2 - \sqrt{2}A_3 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{2}\tilde{A}_3 \,, \qquad & m_3^2 = \frac{4}{\pi^3} \\ -A_1 - A_2 + \sqrt{2}A_3 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{2}\tilde{A}_3 \,, \qquad & m_5^2 = \frac{20}{3\pi^3} \,. \end{aligned} \tag{4.13}$$

The two massive states are the anomalous $U(1)$ which have been found in the spectrum of the original six-dimensional Z_2 orientifold. The indices are taken from the four-dimensional counting and denote which masses are affected by six-dimensional anomalies. Notice that the linear combinations agree with our expectations.

Another interesting limit of the Z_6' orientifold is $\mathcal{V}_3 \to 0$. In this limit, the two linear combinations that are free of four-dimensional anomalies become massless. This is consistent with the fact that the six-dimensional anomalies which are responsible for their masses cancel locally in this limit.

4.3 The four-dimensional Z_6 orientifold

The orbifold rotation vector is $(v_1, v_2, v_3) = (1, 1, -2)/6$. Since there is an order two twist ($k = 3$), we have one set of $D5$-branes that are stretched in the 4D Minkowski and wrap the third torus T_3^2. Tadpole cancellation implies the existence of 32 $D9$-branes and 32 $D5$-branes that we put together at one of the fixed points of the Z_2 action (namely the origin). The Chan-Paton 'shift' vectors are

$$V_{5,9} = \frac{1}{12}(1,1,1,1,1,1,5,5,5,5,5,5,3,3,3,3) \,. \tag{4.14}$$

The gauge group has a factor of $\mathrm{U}(6) \times \mathrm{U}(6) \times \mathrm{U}(4)$ coming from the $D9$-branes and an isomorphic factor coming from the $D5$-branes. This orientifold has different supersymmetries in different sectors. The $N = 1$ sectors correspond to $k = 1, 2, 4, 5$, while for $k = 3$ we have $N = 2$ sectors.

The four-dimensional anomalies of the U(1)s have been computed in [20] and the anomaly matrix is

$$A_{QTT} \sim \begin{pmatrix} 6 & -3 & \sqrt{6} & 3 & 0 & \sqrt{6} \\ 3 & -6 & -\sqrt{6} & 0 & -3 & -\sqrt{6} \\ -9 & 9 & 0 & -3 & 3 & 0 \\ 3 & 0 & \sqrt{6} & 6 & -3 & \sqrt{6} \\ 0 & -3 & -\sqrt{6} & 3 & -6 & -\sqrt{6} \\ -3 & 3 & 0 & -9 & 9 & 0 \end{pmatrix} \tag{4.15}$$

there are three linear combinations that are free of anomalies: $A_1 + A_2 - \sqrt{3/2}\, A_3$, $\tilde{A}_1 + \tilde{A}_2 - \sqrt{3/2}\, \tilde{A}_3$ and $A_3 - \tilde{A}_3$.

The contributions to the mass matrix [16] are:

$$\begin{aligned} \frac{1}{2} M^2_{aa,ij} &= -\frac{\sqrt{3}}{48\pi^3} \Big(\mathrm{Tr}[\gamma_1 \lambda_i^a]\, \mathrm{Tr}[\gamma_1 \lambda_j^a] + \mathrm{Tr}[\gamma_5 \lambda_i^a]\, \mathrm{Tr}[\gamma_5 \lambda_j^a] + \\ &\qquad + 3(\mathrm{Tr}[\gamma_2 \lambda_i^a]\, \mathrm{Tr}[\gamma_2 \lambda_j^a] + \mathrm{Tr}[\gamma_4 \lambda_i^a]\, \mathrm{Tr}[\gamma_4 \lambda_j^a]) \Big) - \\ &\quad - \frac{\mathcal{V}_3}{3\pi^3}\, \mathrm{Tr}[\gamma_3 \lambda_i^a]\, \mathrm{Tr}[\gamma_3 \lambda_j^a] \end{aligned} \tag{4.16}$$

for $a = 5, 9$, while

$$\begin{aligned} \frac{1}{2} M^2_{59,ij} &= -\frac{\sqrt{3}}{48\pi^3} \Big(\mathrm{Tr}[\gamma_1 \lambda_i^5]\, \mathrm{Tr}[\gamma_1 \lambda_j^9] + \mathrm{Tr}[\gamma_5 \lambda_i^5]\, \mathrm{Tr}[\gamma_5 \lambda_j^9] + \\ &\qquad + 3(\mathrm{Tr}[\gamma_2 \lambda_i^5]\, \mathrm{Tr}[\gamma_2 \lambda_j^9] + \mathrm{Tr}[\gamma_4 \lambda_i^5]\, \mathrm{Tr}[\gamma_4 \lambda_j^9]) \Big) - \\ &\quad - \frac{\mathcal{V}_3}{12\pi^3}\, \mathrm{Tr}[\gamma_3 \lambda_i^5]\, \mathrm{Tr}[\gamma_3 \lambda_j^9]\,. \end{aligned} \tag{4.17}$$

Notice that the $N = 2$ sector contributes with a term proportional to $\mathcal{V}_3$. The mass matrix of the anomalous U(1)s has the following eigenvalues and eigenstates [16]:

$$m_1^2 = 0\,, \qquad A_1 + A_2 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{6}(A_3 - \tilde{A}_3)\,; \tag{4.18}$$

$$m_2^2 = \frac{3\sqrt{3}}{2}\,, \qquad A_1 - A_2 - \tilde{A}_1 + \tilde{A}_2\,; \tag{4.19}$$

$$m_3^2 = 3\sqrt{3}\,, \qquad A_1 - A_2 + \tilde{A}_1 - \tilde{A}_2\,; \tag{4.20}$$

$$m_4^2 = 8\mathcal{V}_3\,, \qquad -\sqrt{\frac{3}{2}}(A_1 + A_2 - \tilde{A}_1 - \tilde{A}_2) - A_3 + \tilde{A}_3\,; \tag{4.21}$$

$$m_{5,6}^2 = \frac{7\sqrt{3} + 80\mathcal{V}_3 \pm \beta}{12}\,, \qquad \frac{40\mathcal{V}_3 - \sqrt{3} \pm \beta}{12\sqrt{2} - 40\sqrt{6}\mathcal{V}_3}(A_1 + A_2 + \tilde{A}_1 + \tilde{A}_2) + A_3 + \tilde{A}_3\,; \tag{4.22}$$

where $\beta = \sqrt{147 - 1040\sqrt{3}\mathcal{V}_3 + 6400\mathcal{V}_3^2}$. In the limit $\mathcal{V}_3 \to 0$ the m_4, m_6 become zero. This is the consequence of the local cancellation of the six-dimensional anomalies in this limit.

4.4 Decompactification of the Z_6 orientifold

In the Z_6 orientifold, the $\alpha_1^i, a_2^i, a_4^i, \alpha_5^i$ axions are living in the 4D Minkowski space, and the α_3^i in 4D Minkowski space plus the third torus T_3.

The decompactification limits of the first and second tori ($\mathcal{V}_1, \mathcal{V}_2 \to \infty$) do not have any special interest since none of the fields becomes six-dimensional and there are no six-dimensional anomalies.

4.4.1 Decompactification of the third torus ($\mathcal{V}_3 \to \infty$)

If we decompactify the third torus ($\mathcal{V}_3 \to \infty$), all the string states from the $k = 3$ sector and the a_3^i axions become six-dimensional. The rest of the sectors and axions remain four-dimensional and do not contribute to six-dimensional anomalies. The new gauge group can be found by the action of the γ_3 on the Chan-Paton. The orbifold rotation $3(v_1, v_2) = (1, -1)/2$ shows that D5-branes survive in this limit. The 'shift' vector is now $3V_a$ where V_a is given in(4.14). The old $\mathrm{U}(6) \times \mathrm{U}(6) \times \mathrm{U}(4)$ gauge group is enhanced to $\mathrm{U}(16)$, which is the gauge group of the Z_2 six-dimensional orientifold (table 1). The generators are combined as $T_{\mathrm{U}(16)} \sim T_{\mathrm{U}(6)_1} \oplus T_{\mathrm{U}(6)_2} \oplus \overline{T}_{\mathrm{U}(4)}$. Therefore, we can determine how the old spectrum is enhanced to the new one. The $(\bar{6}, 1, \bar{4})$, $(1, 6, 4)$ and $(6, \bar{6}, 1)$ combine in the adjoint of $\mathrm{U}(16)$. The $(15, 1, 1)$, $(1, 6, \bar{4})$ are in the antisymmetric 120 and $(1, \overline{15}, 1)$, $(\bar{6}, 1, 4)$ in the $\overline{120}$.

By the way that the generators of the $\mathrm{U}(6)^2 \times \mathrm{U}(4)$ are enhanced to the $\mathrm{U}(16)$ we can expect that the six-dimensional $\mathrm{U}(1)$ gauge boson of the $\mathrm{U}(16)$ will be a linear combination $A_1 + A_2 - \sqrt{\frac{2}{3}} A_3$ where the normalization coefficient in front of A_3 takes into account the difference of the rank. Similarly for the tilde.

The contributions of the six-dimensional anomalies to the four-dimensional mass generation are given by the $\mathcal{V}_3 \to \infty$ limit in (4.16), (4.17). We find (for $a = 5, 9$):

$$\frac{1}{2} M_{aa,ij}^2 = -\frac{1}{3\pi^3} \mathrm{Tr}[\gamma_3 \lambda_i^a] \, \mathrm{Tr}[\gamma_3 \lambda_j^a] \tag{4.23}$$

while, for twisted open strings:

$$\frac{1}{2} M_{59,ij}^2 = -\frac{1}{12\pi^3} \mathrm{Tr}[\gamma_3 \lambda_i^5] \, \mathrm{Tr}[\gamma_3 \lambda_j^9] \tag{4.24}$$

which are the same (upon normalization) as the contributions of the six-dimensional generation of the Z_2 orientifold (section 3.1.1). The mass-matrix has four zero eigenvalues, with eigenvectors: $\sqrt{\frac{2}{3}} \tilde{A}_1 + \tilde{A}_3$, $-\tilde{A}_1 + \tilde{A}_2$, $\sqrt{\frac{2}{3}} A_1 + A_3$, $-A_1 + A_2$ and two massive states with eigenvalues:

$$\begin{aligned} &A_1 + A_2 - \sqrt{\frac{2}{3}} A_3 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{\frac{2}{3}} \tilde{A}_3 \,, \qquad M_4^2 = \frac{4}{\pi^3} \\ &A_1 + A_2 - \sqrt{\frac{2}{3}} A_3 + \tilde{A}_1 + \tilde{A}_2 - \sqrt{\frac{2}{3}} \tilde{A}_3 \,, \qquad m_5^2 = \frac{20}{3\pi^3} \,. \end{aligned} \tag{4.25}$$

The two massive states are the anomalous $\mathrm{U}(1)$s which have been found in the spectrum of the original six-dimensional Z_2 orientifold. It is easy to verify that the four-dimensional massless state $A_1 + A_2 - \tilde{A}_1 - \tilde{A}_2 + \sqrt{6}(A_3 - \tilde{A}_3)$ (4.22) is still massless in six dimensions.

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5. Conclusions

In this paper we have shown that four-dimensional non-anomalous U(1)s can become massive if in decompactification limits they suffer from six-dimensional anomalies.

We have studied several four-dimensional orientifolds. In the decompactification limit, there are sectors in such orientifolds that become six dimensional. The original four-dimensional massless spectrum combined with Kaluza-Klein states that become massless in this limit, enhance to the massless spectrum of six-dimensional orientifolds. Some RR axions also become six-dimensional fields.

In the six-dimensional orientifolds, we have calculated the stringy anomalous U(1) masses that are in accordance with six-dimensional anomalies. The six-dimensional RR axions contribute to the mass-generation of the anomalous U(1)s through the Green-Schwarz mechanism.

We verified that the six-dimensional mass-matrix is the same as the volume dependant contribution to the four-dimensional matrix. Thus, six-dimensional anomalies play indirectly a role in four-dimensional masses and explain why some non-anomalous U(1) gauge bosons have a non-zero mass.

Our analysis has direct implications for model building both in string theory and field theory orbifolds. It provides a necessary and sufficient condition for a non-anomalous U(1) to remain massless (the hypercharge for example). One has just to check the associated higher dimensional anomalies in the various decompactification limits.

The masses of the anomalous U(1)s are always as heavy or lighter than the string scale. Therefore, production of these new gauge bosons in particle accelerators provides both constrains on model building and new potential signals at colliders, if the string scale is around a few TeV.

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A. Definitions and identities

The Dedekind function is defined by the usual product formula (with $q = e^{2\pi i\tau}$)

$$\eta(\tau) = q^{\frac{1}{24}} \prod_{n=1}^{\infty} (1 - q^n)\,. \tag{A.1}$$

The Jacobi ϑ-functions with general characteristic and arguments are

$$\vartheta[^{\alpha}_{\beta}](z|\tau) = \sum_{n\in Z} e^{i\pi\tau(n-\alpha/2)^2} e^{2\pi i(z-\beta/2)(n-\alpha/2)}\ . \tag{A.2}$$

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The $\vartheta[^1_1]$ is an odd function whose first derivative at zero is $\vartheta[^1_1]'(0|\tau) = 2\pi\eta^3$. Some modular properties of these functions are provided:

$$\eta\left(\frac{-1}{\tau}\right) = \sqrt{-i\tau}\eta(\tau)\,, \qquad \vartheta\begin{bmatrix}\alpha\\\beta\end{bmatrix}\left(\frac{z}{\tau}\Big|\frac{-1}{\tau}\right) = \sqrt{-i\tau}\; e^{i\pi\left(\frac{\alpha\beta}{2}+\frac{z^2}{\tau}\right)}\vartheta\begin{bmatrix}\beta\\-\alpha\end{bmatrix}(z|\tau)\,. \tag{A.3}$$

A very useful identity is

$$\sum_{\alpha,\beta=0,1}\eta_{\alpha\beta}\,\vartheta\begin{bmatrix}\alpha\\\beta\end{bmatrix}(v)\prod_{i=1}^{3}\vartheta\begin{bmatrix}\alpha+h_i\\\beta+g_i\end{bmatrix}(0) = -\vartheta_1\left(\frac{-v}{2}\right)\prod_{i=1}^{3}\vartheta\begin{bmatrix}1-h_i\\1-g_i\end{bmatrix}\left(\frac{v}{2}\right) \tag{A.4}$$

valid for $\sum h_i = \sum g_i = 0$.

B. Correlation functions on the annulus

We present here the derivation of the propagators that we will use for the calculation of the annulus $\mathcal{A}$. This surface can be defined as quotient of the torus $\mathcal{T}$ under the involution [13]

$$\mathcal{I}_{\mathcal{A}}(z) = 1-\bar{z}\,. \tag{B.1}$$

Thus, the correlators can be expressed in terms of the propagators on the torus. For the bosonic case we have

$$\langle X(z)X(w)\rangle_{\mathcal{T}} = -\frac{1}{4}\log\left|\frac{\vartheta_1(z-w|\tau)}{\vartheta_1'(0|\tau)}\right|^2 + \frac{\pi(z_2-w_2)^2}{2\tau_2} \equiv P_B(z,w) \tag{B.2}$$

and symmetrizing under the involution:

$$\begin{aligned}\langle X(z)X(w)\rangle_{\mathcal{A}} &= \frac{1}{2}[P_B(z,w)+P_B(\mathcal{I}_{\mathcal{A}}(z),w)+P_B(z,\mathcal{I}_{\mathcal{A}}(w))+P_B(\mathcal{I}_{\mathcal{A}}(z),\mathcal{I}_{\mathcal{A}}(w)]\\ &= P_B(z,w)+P_B(z,1-\bar{w})\,.\end{aligned} \tag{B.3}$$

In the amplitude, the partial derivative of the above correlator (B.3) appears. Thus, we give the expression that we use for $w = 1/2$:

$$\left\langle \partial_z X(z) X\left(\frac{1}{2}\right)\right\rangle_{\mathcal{A}} = -\frac{1}{2}\left[\partial_z\log\vartheta_1\left(z-\frac{1}{2}\Big|\tau\right)+\frac{2\pi i z_2}{\tau_2}\right] \tag{B.4}$$

for $z = z_1 + iz_2$. We remind also that $\partial_z = (\partial_{z_1} - i\partial_{z_2})/2$.For the fermionic correlators on the torus we have the identity:

$$\langle\psi(z)\psi(w)\rangle^2\begin{bmatrix}\alpha\\\beta\end{bmatrix} = -\frac{1}{4}\mathcal{P}(z-w) - \pi i\partial_\tau\log\frac{\vartheta\begin{bmatrix}\alpha\\\beta\end{bmatrix}(0|\tau)}{\eta(\tau)} \tag{B.5}$$

where $\mathcal{P}(z-w)$ is the Weierstrass function. Symmetrizing the torus propagator under the involution we find that (B.5) holds also for the annulus.

C. Computations in type-I orientifolds

In the appendix, we give some more details about the computations of the mass term.

C.1 Open strings attached on the same kind of branes

The internal partition function of strings attached on the same kind of branes is:

$$Z^{aa}_{int,k}[^{\alpha}_{\beta}] = \prod_{j=1}^{2}(-2\sin\pi k v_j)\frac{\vartheta[^{\alpha}_{\beta+2kv_j}](0|\tau)}{\vartheta[^{1}_{1+2kv_j}](0|\tau)} \quad \text{for a=5,9}\,. \tag{C.1}$$

After the use of (A.4) and the fact that $\vartheta[^1_1](0|\tau)=0$, we find for the annulus amplitude:

$$\begin{aligned}\mathcal{A}^{aa}_k &= -\frac{1}{2N}\int[d\tau]\tau_2^{1+\delta/2}[2\pi\eta^3(\tau)]^{\delta}\left[\frac{1}{2\pi\tau^3}4\sin^2\frac{\pi k}{N}\right]\\ &= -\frac{(2\pi)^{\delta}}{\pi N}\sin^2\frac{\pi k}{N}\int_0^{i\infty}d\tau_2\tau_2^{-2+\delta/2}\eta^{3\delta}(\tau_2)\,.\end{aligned} \tag{C.2}$$

We are interested in the UV limit of the above integral. The annulus moduli is $\tau_2 = it/2$:

$$\begin{aligned}\mathcal{A}^{aa,UV}_k &= -\frac{(2\pi)^{\delta}}{\pi N}\sin^2\frac{\pi k}{N}2^{1-\delta/2}\int_0^1 dt\ t^{-2+\delta/2}\eta^{3\delta}(it/2)\\ &= -\frac{(2\pi)^{\delta}}{\pi N}\sin^2\frac{\pi k}{N}2^{1-\delta/2}\int_0^1 dt\ t^{-2+\delta/2}\left[\left(\frac{2}{t}\right)^{1/2}\eta\left(\frac{2}{t}\right)\right]^{3\delta}\\ &= -\frac{4}{\pi^2\delta N}\left(\frac{8}{\delta}\right)^{\delta}\sin^2\frac{\pi k}{N}\,\Gamma\left(1+\delta,\frac{\pi\delta}{2}\right).\end{aligned} \tag{C.3}$$

where $\Gamma(a,x)$ is the incomplete Γ-function and $\Gamma(1,0)=1$.

C.2 Open strings attached on different kind of branes

Strings attached on different kind of branes have coordinates X^a with mixed Dirichlet-Neumann boundary conditions. Those coordinates are half-integer moded and there are no windings or momenta. The fermionic sectors interchange modes between R and NS (since the R states should have same modes than the coordinates) keeping the total fermionic pact unchanged. Thus, the internal partition function for such strings is:

$$Z^{59}_{int,k}[^{\alpha}_{\beta}] = \prod_{j=1}^{2}\frac{\vartheta[^{\alpha+1}_{\beta+2kv_j}](0|\tau)}{\vartheta[^{0}_{1+2kv_j}](0|\tau)}\,. \tag{C.4}$$

Following the same procedure, like in the case of the strings with the same boundary conditions, we substitute (C.4) in (3.8) and after a bit of "thetacology" we find:

$$\mathcal{A}^{59}_k = -\frac{1}{2N}\int[d\tau]\tau_2^{1+\delta/2}[2\pi\eta^3(\tau)]^{\delta}\left[\frac{1}{2\pi\tau^3}\right]. \tag{C.5}$$

The integral is the same as in the case of the strings having the same boundary conditions. Using the above result we find:

$$\mathcal{A}^{59,UV}_k = -\frac{1}{\pi^2\delta N}\left(\frac{8}{\delta}\right)^{\delta}\Gamma\left(1+\delta,\frac{\pi\delta}{2}\right). \tag{C.6}$$

D. The anomaly-free massless spectrum of the $N = 1$ six-dimensional orientifolds

In the next sections, we will show that the spectrum of the N=1 six-dimensional orientifolds does not suffer from irreducible non-abelian anomalies.

We reevaluate the spectrum of these models using the 'shift' vectors that are given for each model. We were especially careful in distinguishing the representations from the conjugate representations since it was not clear in the previous literature. Our results are provided in table 1. In this section we will prove that the spectrum does not suffer from irreducible non-abelian anomalies [23, 24].

Before we continue to the computations, we review the Z_N orientifolds. The Ramond sector $|s_1 s_2 s_3 s_4, ij\rangle\lambda_{ji}$ is constrained by GSO projection to have even number of minus signs between the s_i. The orbifold acts on the states with $a_N^k = e^{\frac{2\pi i k}{N}(J_{67}-J_{89})}$.

For all the orientifolds Z_N with $Z \neq 2$, in the vector multiplet there are two fermionic fields with $s_1 = s_2 = \pm 1/2$, that transform like $(2,1)$ under the space-time $\mathrm{SO}(4) = \mathrm{SU}(2)\times \mathrm{SU}(2)$. However, the hypermultiplets contain two spinors $(1,2)$ that one is conjugate of the other under the gauge group. In the $N = 6$ for example, the spinors of 99 and 55 transform like:

$$\begin{aligned} s_1 &= -s_2,\ s_3 = -s_4 = +\frac{1}{2} \qquad (6,1,1),\ (\bar{4},1,8),\ (1,4,\bar{8}),\ (1,\bar{6},1) \\ s_1 &= -s_2,\ s_3 = -s_4 = -\frac{1}{2} \qquad (\bar{6},1,1),\ (4,1,\bar{8}),\ (1,\bar{4},8),\ (1,6,1) \end{aligned} \tag{D.1}$$

and for 59 string-states we have one fermionic state in $(1,2)$:

$$\begin{aligned} s_1 = -s_2 \qquad & (4,1,1;\overline{4},1,1) + (\overline{4},1,1;4,1,1) + \\ & +(1,4,1;1,\overline{4},1) + (1,\overline{4},1;1,4,1) + \\ & +(1,1,8;1,1,\overline{8}) + (1,1,\overline{8};1,1,8)\,. \end{aligned} \tag{D.2}$$

Thus, the six-dimensional non-abelian anomalies vanish if we use the relations between the quartic Casimir of $\mathrm{SU}(N)$ in various representations. We need also [25]:

$$\begin{aligned} \mathrm{Tr}[T^4]_{adj} &= 2N\ tr[T^4]_{\square} + 6\ tr[T^2]^2_{\square} \\ \mathrm{Tr}[T^4]_{\begin{smallmatrix}\square\\\square\end{smallmatrix}} &= (N-8)\ tr[T^4]_{\square} + 3\ tr[T^2]^2_{\square}\,. \end{aligned} \tag{D.3}$$

We will cancel the $\mathrm{Tr}[F^4]$, non-abelian anomalies for each orientifold separately.

D.1 Z_2 orbifold

The Z_2 orbifold in the type-I string theory, gives $\mathrm{U}(16)_9\times\mathrm{U}(16)_5$ gauge group. The massless states are given in table 1. The spectrum does not suffer from non-abelian anomalies. The contribution of all the spinors in the $\mathrm{Tr}[F^4]$ anomaly of $\mathrm{SU}(16)_9$ is:

$$2(-2\cdot 16)tr[T^4] + 2\cdot 2(16-8)tr[T^4] + (16+16)tr[T^4] = 0 \tag{D.4}$$

The first term is coming from the $2(1,2)$ spinors of the adjoint. The second term is coming from the two $(2,1)$ that (only in the Z_2 case) transform similarly, like $120 + \overline{120}$. The last term is coming from the $(1,2)$ state (the 59 string states). Similarly, we can show that the $\mathrm{Tr}[F^4]$ anomaly of the $\mathrm{SU}(16)_5$ vanish too.

D.2 Z_3 Orbifold

In the type-I Z_3 orbifold, there are only $D9$-branes, characterized by the U(8) $\times$ SO(16) gauge group. The vanishing of the non-abelian anomalies is strait-forward:

$$\begin{aligned} \mathrm{SU}(8) \quad &: \quad 2(-2\cdot 8)tr[T^4] + 2(8-8)tr[T^4] + (16+16)tr[T^4] = 0 \\ \mathrm{SO}(16) \quad &: \quad -2(16-8)tr[T^4] + (8+8)tr[T^4] = 0\,. \end{aligned} \tag{D.5}$$

The first, second and third term for the SU(8) are the contribution of the adjoint, antisymmetric and bifundamental of the spectrum. The first and second terms for the SO(16) are the adjoint and bifundamentals respectively.

D.3 Z_4 Orbifold

In the type-I Z_4 orbifold, there are 32 $D9$ and 32 $D5$-branes, characterized by the $\mathrm{U}(8)_9 \times \mathrm{U}(8)_9 \times \mathrm{U}(8)_5 \times \mathrm{U}(8)_5$ gauge group. The contribution of all the spinors to the $\mathrm{Tr}[F^4]$ anomaly of one of the SU(8) is:

$$2(-2\cdot 8)tr[T^4] + 2(8-8)tr[T^4] + (8+8+8+8)tr[T^4] = 0 \tag{D.6}$$

The first, second and third term are the contribution of the adjoint, antisymmetric and bifundamental of the spectrum. The coefficient have been explained before.

D.4 Z_6 Orbifold

In the type-I Z_6 orbifold, there are 32 $D9$ and 32 $D5$-branes characterized by the U(4) $\times$ U(4)$\times$U(8) gauge group for the $D9$-branes and a isomorphic gauge group for the $D5$-branes. The contribution of all the spinors in the anomaly of one of the SU(4) is:

$$2(-2\cdot 4)tr[T^4] + 2(4-8)tr[T^4] + (8+8+4+4)tr[T^4] = 0\,. \tag{D.7}$$

The first, second and third term are the contribution of the adjoint, antisymmetric and bifundamental of the spectrum. The contribution in the anomaly of one of the SU(8) is:

$$2(-2\cdot 8)tr[T^4] + (4+4+4+4+8+8)tr[T^4] = 0\,. \tag{D.8}$$

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